

How many stocks are enough for diversifying Canadian institutional portfolios?

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Abstract

Portfolio risk is a function of the number of stocks held in portfolios. We simulate portfolios using daily observations for all traded and delisted equities in Canada from 1975 to 2011 and we calculate several measures of risk, including heavy-tailed to account for black swan events. For each risk measure, we calculate the average number of portfolio holdings and the upper limits of these holdings to assure investors of a specific reduction in diversifiable risk. In contrast to previous literature that suggests 10-15 stocks are enough to provide adequate diversification for an average investor, we find that in fact more than 50 stocks are needed to achieve the same level of diversification most of the time instead of on average.

Keywords: Portfolio diversification, heavy tailed risk, expected shortfall, time series standard deviation, terminal wealth standard deviation, Canadian equities, institutional investors.

1. Introduction

In this paper, we study the question of optimal portfolio diversification for Canadian institutional investors between 1975 and 2011. We determine the number of stocks in a portfolio¹ required to minimize diversifiable risk. Using daily data, we calculate various measures of risk, including some that account for black swan events. Previous research has analyzed the optimal portfolio sizes for an average investor (Solnik (1974) suggests holding 10-15 stocks, Copp and Cleary (1999) - 30-50 stocks and Kryzanowski and Singh (2010) - 20-25 stocks). We build on our predecessors' contributions by estimating confidence bands around the average number of stocks in portfolios that provide this level of diversification 90% of the time as opposed to achieving this diversification level on average. The portfolio size based on this 90% confidence band will insure that the desired level of diversification is achieved in 90% of the cases, instead of 50% of the time. Holding 19 to 25 stocks² will provide investors with an adequate level of diversification

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¹Hereafter, referred to as portfolio size.

²See Table 1 last row.

only on average. However, the same size recommendation may not provide this level of diversification for half of the investors. Comparing the year-by-year dynamic of portfolio holdings, we find that through time, the recommended number of stocks is affected by the state of the markets - distressed or quiescent, as well as by average correlations among stocks in the Canadian market.

Most academic literature argues that if diversifiable risk can be eliminated with a relatively small number of stocks, equity funds cannot justify holding an excessive number of stocks, which result in over-diversification and exaggerated fund fees. The same literature favours smaller portfolio sizes (about 10 to 25 stocks) to achieve a well-diversified portfolio. However, the portfolio sizes of most institutional investors are in excess of these recommendations.³ Holding too many stocks is costly both in terms of transaction costs as well as the opportunity cost of monitoring large diversified portfolios where associated fees dampen overall performance. Holding too few stocks exposes the fund to avoidable firm-specific risk. Previous academic research relies on averages and does not take into account the chance of a particular fund or institutional investor falling below the average, i.e. the risk of not achieving a specific risk target. Most institutional investors would feel more comfortable if they could hold a portfolio that would be well-diversified 90% of the time rather than 50% of the time. We find that for the purpose of diversification, portfolios held by Canadian institutional investors should be larger than those suggested in previous academic literature. In the U.S., firm specific risk has grown over the past thirty years relative to the overall variability of the stock market, while correlations between stocks have correspondingly decreased (see [Campbell, Lettau, Malkiel, and Xu, 2001](#)). This reinforces the advisability of larger portfolios. We check whether this result holds for Canada too.

We simulate random portfolios based on actual daily Canadian equity returns over the period 1975 to 2011. We construct equally-weighted random portfolios⁴, each of different size, ranging from portfolios consisting only of one security to a broad market portfolio including all actively traded securities at the time. For each of these different-sized portfolios and each year we calculate time series standard deviation (SD), 1% expected shortfall (ES) and terminal wealth standard deviation (TWSD).⁵ We focus on SD as our benchmark to be able to compare our results to the previous literature. The $ES_{1\%}$ is a downside risk measure that accounts for black swan events and is associated with the lowest 1% of the return distribution. The TWSD is a standard

³For example, in 2013, excluding the smallest 25% and the biggest 25% out of a total of 2,088 all-domestic US equity mutual funds results in a range of 49 to 129 stocks. Source: Morningstar Fund Screener.

⁴In contrast to our approach [Foerster, Fogler, and Sapp \(2011\)](#) investigate the size effect on Canadian portfolios by concentrating on small cap stocks. Further diversification benefits, however, may be achieved by stratified sampling across market capitalizations.

⁵We define the portfolio standard deviation as $\sigma = \sqrt{\sum_{t=1}^T \frac{(r_t - \bar{r})^2}{T-1}}$, where r is portfolio return. We estimate expected shortfall from the portfolio returns. Let $r_{t,\alpha}$ be the empirical α th quantile. We define $ES_\alpha = -\frac{1}{T_\alpha} \sum_{t=1}^T r_t \mathbf{1}(r_t \leq r_{t,\alpha})$, where $\mathbf{1}(\cdot) = 1$ if $r_t \leq r_{t,\alpha}$ and 0 otherwise, and T_α denotes the number of r_t no greater than $r_{t,\alpha}$. Terminal wealth of a portfolio is defined as $TW = \sum_{t=1}^T r_t$ and terminal wealth standard deviation can be expressed as $TWSD = \sqrt{\sum_{i=1}^M \frac{(TW_i - \overline{TW})^2}{M-1}}$, where $\overline{TW} = \sum_{i=1}^M \frac{TW_i}{M}$ is the average terminal wealth over M random portfolios. Thus, the TWSD risk measure accounts for the volatility of terminal wealth.

benchmark measure for buy-and-hold no-rebalancing portfolios, typically suitable for pension funds. We trace the dynamics of diversification benefits over the past 37 years in the Canadian market.⁶

We find that for institutional investors, especially pension funds (relying on TWSD), who seek to avoid large losses in extreme market events, recommended portfolio sizes are typically larger than those recommended to investors concerned with general deviation from the trend (based on SD as a risk measure). The 90% confidence bands around our average risk measures give us an upper limit to the number of stocks required in a portfolio that assures 90% of the time a 90% risk reduction. When measuring extreme losses with $ES_{1\%}$, we find that, over the period 1975-2011, portfolios of 41 stocks would provide sufficient diversification, but when TWSD is used as a risk measure 61 stocks are required (see Table 1). However, to achieve the same level of risk reduction but only on average (instead of 90% of the time), we find that portfolios are typically smaller (25 stocks relying on SD as a risk measure and 19 stocks based on $ES_{1\%}$).

In Section 2, we discuss our approach and the data. In Section 3, we present our results. We conclude in Section 4.

2. Data and Methodology

We assume that each portfolio is comprised of long positions on common stocks only. Our data, from Thomson Reuters Datastream, consist of daily total returns on common stocks listed on the Toronto Stock Exchange (TSX) from 1975 to 2011. To avoid survivorship bias we collect the data for both active and delisted stocks. For each of these years, we consider only stocks that have traded at least 75% of the trading days in a particular year. This is done to avoid unreasonably low correlations of some thinly-traded stocks with the rest of portfolio holdings.

We model portfolio total risk as a combination of systematic market risk and firm-specific risk. As the number of stocks included in a portfolio approaches the number of stocks in the market, portfolio risk approaches the level of market risk. Reduction in portfolio risk can be achieved up to the point where all firm-specific risk has been eliminated or where the incremental decrease in firm-specific risk brings no real benefit. It is generally accepted that 90% reduction in diversifiable risk yields well-diversified portfolios.

We construct portfolios by randomly drawing n stocks without replacement from all available stocks on the TSX each year. We use equal weights to construct all of our portfolios. We simulate 10,000 random portfolios unless the number of combinations of n stocks out of all available stocks in the market, N , is lower. For example, when $n = 1$, the number of unique single security portfolios equals the total number of stocks available on the market⁷ and when $n = N$ only one unique equally-weighted portfolio can be constructed - we define it as the market portfolio. We find that 10,000 replications are enough to provide us with stable estimates of the mean, median and the 90% percentile of our risk measures.

⁶In Alexeev and Tapon (2012) four additional markets are discussed at length.

⁷The number of actively traded stocks on the TSX has steadily increased from 1975 (107 stocks) to 2011 (1,173 stocks). It is interesting to note that the peak number of actively traded stocks on the TSX was in 2000 (1,264 stocks) and in 2007 (1,370).

Investment professionals agree that measuring risk is more contentious than measuring return. For this reason we consider several measures of risk. Our first measure is the time series standard deviation (SD), a well accepted measure of risk. Concentrating on negative returns, downside risk measures account for deviations below a certain threshold, unlike standard deviation, where positive and negative deviations from the expected level are penalized equally. Downside risk measures are particularly useful in accounting for the asymmetries in returns during bull and bear markets. Recent black swan events have shown how important tail risks are. Investment practitioners often use the Value-at-Risk (VaR) measure to incorporate such events. The VaR, however, suffers from a number of problems, the main one being its inability to capture consistently diversification phenomena. For this reason, we use the expected shortfall (ES) measure to account for extreme losses. An additional conceptually different measure, more appropriate for pension fund managers, is terminal wealth standard deviation (TWSD) that assumes buy-and-hold no-rebalancing portfolios during the period and focus only on the volatility of terminal wealth.

3. Results

Knowledgeable investors are aware of the need for a reasonably large number of stocks to minimize diversifiable risk.⁸ Figure 1 displays the dynamic of diversifiable risk remaining for portfolios of various sizes between 1975 and 2011. Using standard deviation as a measure of risk, a 5-stock portfolio results, on average, in a 55% to 70% reduction in diversifiable risk. On the other hand, a 40-stock portfolio exposes an investor to only 3% to 10% of diversifiable risk. Looking at the year 1987 when a major crash occurred, an investor with a 10-stock portfolio would be exposed to 25% of diversifiable risk using standard deviation as a risk measure, but the same 10-stock portfolio would expose this investor to 28% of diversifiable risk using expected shortfall to account for this black swan event. Thus, fund managers more concerned with extreme market events would be exposing themselves to higher levels of diversifiable risk if they wrongly use standard deviation as their point of reference.

As portfolio sizes increase, the percentage of diversifiable risk reduction changes over the years. Conversely, if investors wish to maintain a reduction in diversifiable risk at a fixed 90%, portfolio sizes will vary over time. In Figure 2 we trace the recommended portfolio sizes for three risk measures to achieve a 90% diversifiable risk reduction for an average investor (solid lines). For institutional investors requiring a higher level of assurance (90% of the time instead of on average) we trace the recommended portfolio sizes using two risk measures (dotted lines). For pension fund investors concerned with the variability of terminal wealth, recommendations based on TWSD are traced by a solid line with circles.

In Table 1 and Figure 2, we observe that between 1975 and 1993 the average Canadian investor using SD as the reference should have held 12 to 30 stocks but an institutional investor wishing to reduce diversifiable risk with 90% certainty should have held a larger portfolio (20-43 stocks, in parentheses in Table 1). This is compared to the period 1994 to 2011 when the average investor using SD as a risk measure

⁸According to a Canadian Securities Administrators 2012 survey, 61% of Canadians understand that buying a single company stock does not provide a safer return than a portfolio of stocks.

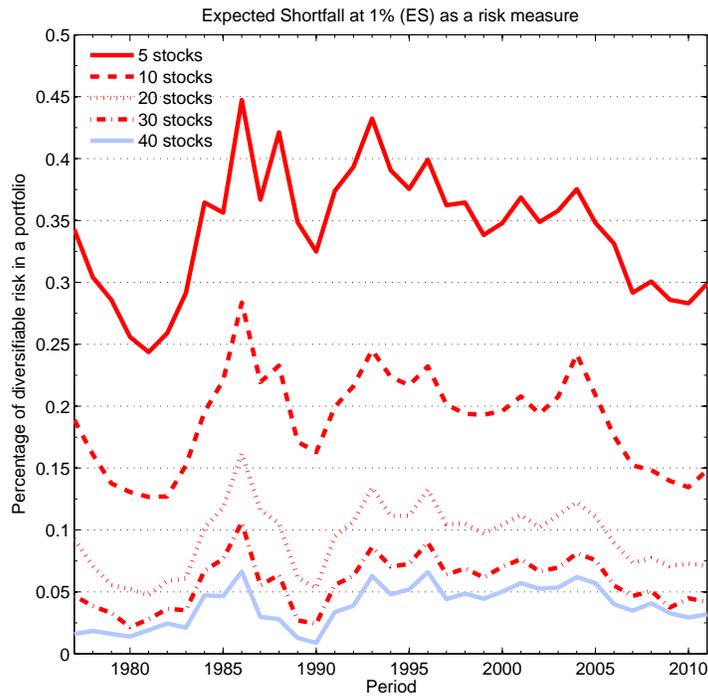
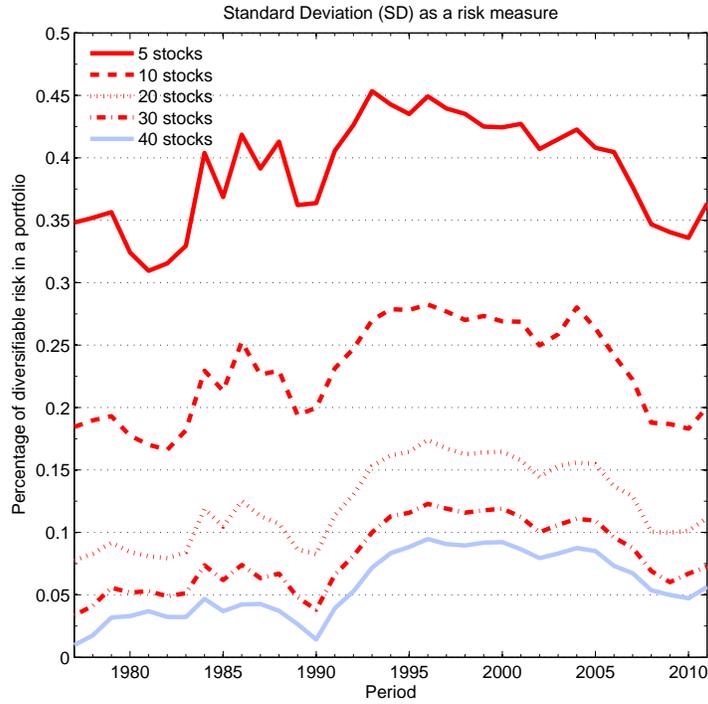


Figure 1: DIVERSIFIABLE RISK REMAINING FOR PORTFOLIOS OF VARIOUS SIZES. The panels above show the dynamic of diversifiable risk remaining for portfolios of various sizes. As the number of stocks in portfolios increases the percentage of diversifiable risk decrease changes over the years.

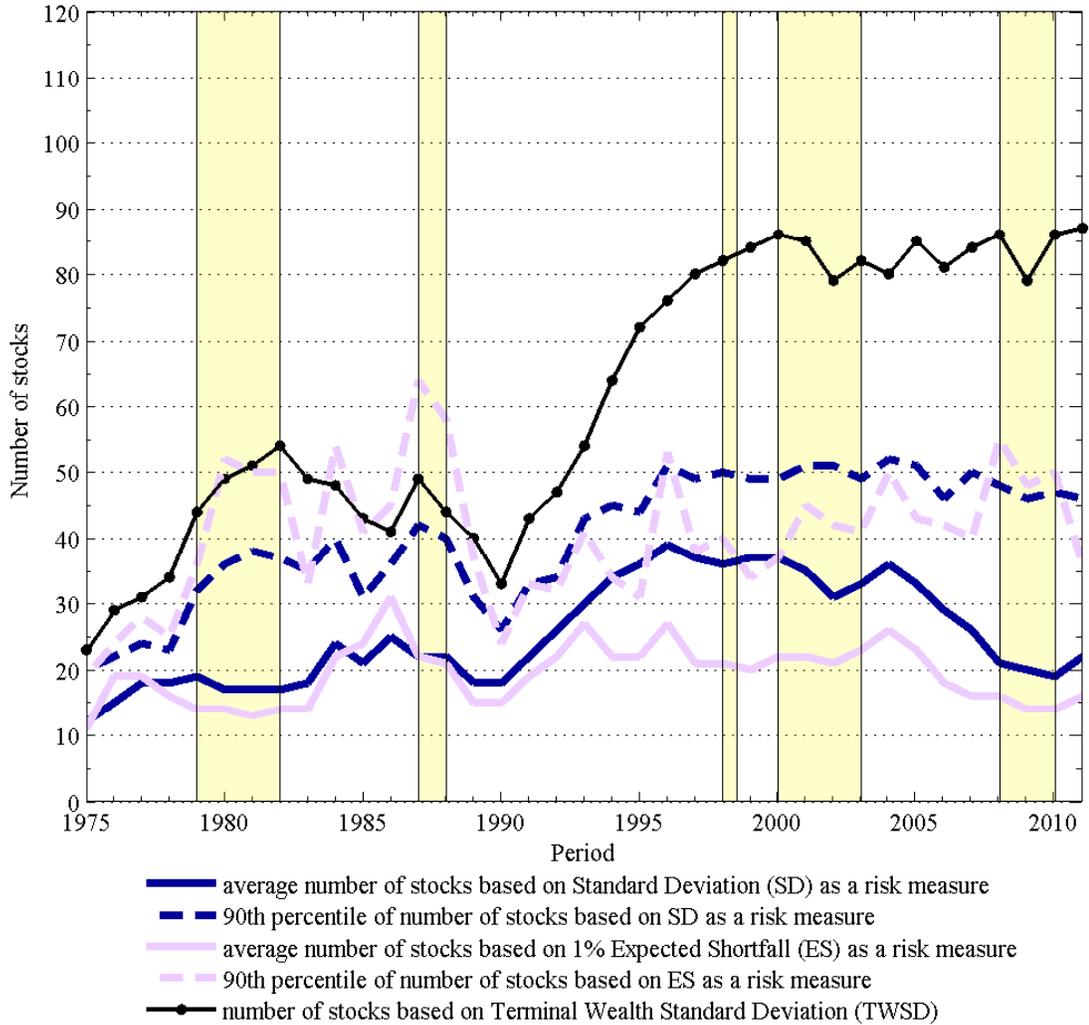


Figure 2: RECOMMENDED PORTFOLIO SIZE TO ACHIEVE 90% REDUCTION IN DIVERSIFIABLE RISK. The solid dark line represents the number of stocks recommended for an average investor to achieve a 90% reduction in diversifiable risk when standard deviation is used as a risk measure. To achieve this reduction 90% of the time, portfolio size is depicted by the dashed dark line. Similarly, for investors concerned with extreme risk and using $ES_{1\%}$ as the risk measure, the portfolio size for an average investor is depicted by the solid light line and the size of the portfolio that assures this reduction 90% of the time is shown by the dashed light line. For investors concerned with terminal wealth standard deviation, our recommended portfolio size is shown by the dark solid line with circles. Shaded regions in the figure represent periods of crises and correspond to events of the 1973 oil crisis (1973-1974), the 1979 oil crisis (1979-1982), Black Monday (1987), the collapse of Long Term Capital Management (LTCM) in 1998, the Dot-com bubble (2000-2002) and the Global Financial Crisis (2008).

would have held 19-39 stocks and an institutional investor (requiring 90% certainty) would have held 44 to 52 stocks. This difference in portfolio sizes between the two periods is a consequence of lower levels of diversifiable risk in the earlier period (observe the difference between market risk and average security risk in Figures 3.A and 3.B) coupled with declining average correlations among stocks (see Figure 3.C between 1975 and 1993).

The slowdown in the Canadian economy between 1992 and 2004 was marked by a wide gap between average security risk and market risk (Figures 3.A and 3.B). During that period we observe the lowest average correlations among stocks (Figure 3.C), and the largest number of stocks required to diversify portfolios (Figure 2 solid lines for SD, ES - except for Black Monday in 1987 - and TWSD). We note a similar trend in our recommendations for institutional investors wishing to diversify 90% of the time instead of 50% of the time (Figure 2 dashed light and dark lines).

The average correlation among stocks between 1975 and 2011 is 0.08 (see Table 2 Panel B). During the prolonged period of poor economic performance between 1982 to 1998 the correlations declined. As we mentioned previously, [Campbell, Lettau, Malkiel, and Xu \(2001\)](#) find a similar trend in average correlations for US equities. This trend, however, reverses after 1999 in the US and Canada. We note that the higher the correlations of stocks with the market, the lower the number of stocks required to diversify portfolios. When large portfolios are recommended (greater than 33 based on SD as the measure of risk) we notice that the associated market volatility and correlations are lower than when recommended portfolios are small (fewer than 18 in Table 2 Panel C). For example, small portfolios of less than 18 stocks are recommended in periods with an average correlation of 0.36 and an increased market volatility of 15.9% (in Table 2 Panel B the average market volatility over the whole period is 13.4%). In contrast, larger portfolios of 33 stocks or more are recommended in periods with an average correlation of 0.18 and a market volatility of 11.1%. In Table 2 Panel D we show the spread between the number of stocks required to assure institutional investors of the desired level of diversification 90% of the time and the portfolio size for the average investor (defined as Δn).⁹ Large spreads ($\Delta n > 19$) are associated with increased market volatility (18.7%) and increased correlations (0.34). Spread of fewer than 11 stocks ($\Delta n < 11$ in Table 2 Panel D) are associated with market volatility of 10.3% and an average correlation of 0.27. We conclude that in periods of high market volatility and large correlations among stocks, institutional investors need to add a larger number of stocks (larger than when markets are quiescent and correlations among stocks are low) to their portfolios to assure the reduction in risk 90% of the time.

In periods of extreme market volatility or market crashes (black swan events), we find that when we use measures of extreme risk such as $ES_{1\%}$, the portfolio size requirements to achieve the desired level of diversification with 90% certainty greatly increase (refer to Table 1, 'Average' row at bottom in parenthesis for the ES: for years 1980-1982 - 50-52 stocks are recommended during the second oil crisis, 41-64 stocks in 1984-1988 when Canada was in a deep recession, 41-50 stocks in years following the bursting of the dot com bubble in 2001-2006 and 48-55 stocks during the Global Financial Crisis in 2008-2010). However, for the same years of extreme market events,

⁹Refer to the difference between the dashed and solid lines in Figure 2.

the recommendations for an average investor based on SD never rise above 36 stocks. Thus, institutional investor, building on these recommendations will hold underdiversified portfolios 50% of the time.

For pension fund managers using TWSD as a risk measure (Table 1 and Figure 2, solid line with circles) the recommended portfolio sizes increase from 23 stocks in 1975 to 87 stocks in 2011.

4. Conclusion

In this study of the size of diversified Canadian institutional portfolios we use daily traded stock returns between 1975 to 2011. We contrast the recommended number of stocks in portfolios of average and institutional investors to achieve a 90% level of diversifiable risk reduction. Our sample period allows us to account for some significant events in Canadian financial market history. We conclude that portfolio size recommendations depend on the particular risk measure, are influenced by market conditions and by correlations among stocks. We identify two types of crises, industry specific meltdowns (2000-2002) and general drops in the market (1987 and 2008-2011). Such crises share a few common characteristics. In the case of market crashes, we note increased market risk measured by SD and especially $ES_{1\%}$ coupled with greatly increased average correlations among securities as well as the correlation of the average security with the market portfolio (see Figure 3).

During market crashes, Figure 2 (solid dark line) shows that the number of stocks required for an average investor to obtain a 90% reduction in diversifiable risk using standard deviation as a measure decreases. At the same time, for institutional investors the number of stocks required to achieve the same level of diversification but with 90% confidence increases. For the average Canadian investor, Table 1 confirms that during major stock market crashes (years 1987 and 2008 marked by significant market volatility spikes in Figures 3.A and 3.B) the number of stocks required to eliminate 90% of diversifiable risk on average was the lowest (21-22 stocks when risk is measured by SD and 16-22 stocks when risk is measured by $ES_{1\%}$). For institutional investors the number of stocks required to achieve the same level of diversification but with 90% confidence are the highest (55 and 64 stocks) during the crises years of 1987 and 2008 when risk is measured as $ES_{1\%}$.

Industry specific meltdowns, such as, arguably, the collapse of Long Term Capital Management and the bursting of the Dot Com bubble, result in highly volatile markets. During these periods, the average correlations among securities and with the market portfolio, were among the lowest (Figure 3.C). Figure 2 shows that more stocks are needed to get the desired level of diversification.

Market conditions will change through time and institutional investors, particularly pension funds, interested in buy-and-hold portfolios for the long haul will require large portfolios, that is, portfolios at the upper end of our portfolio size recommendations. This will ensure that these portfolios will be well diversified under a variety of market conditions. Therefore, our recommendation for institutional investors, is to hold 64 stocks. This will yield a well-diversified portfolio 90% of the time even during the worst market conditions. However, such portfolios will be over-diversified under more normal market conditions. For pension fund managers using TWSD as a standard benchmark measure for buy-and-hold no-rebalancing portfolios, we recommend holding 87 stocks.

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Year	SD	ES	TWSD	Year	SD	ES	TWSD
1975	12 (20)	11 (19)	23	1994	34 (45)	22 (34)	64
1976	15 (22)	19 (24)	29	1995	36 (44)	22 (31)	72
1977	18 (24)	19 (28)	31	1996	39 (51)	27 (53)	76
1978	18 (23)	16 (25)	34	1997	37 (49)	21 (38)	80
1979	19 (32)	14 (36)	44	1998	36 (50)	21 (40)	82
1980	17 (36)	14 (52)	49	1999	37 (49)	20 (34)	84
1981	17 (38)	13 (50)	51	2000	37 (49)	22 (37)	86
1982	17 (37)	14 (50)	54	2001	35 (51)	22 (45)	85
1983	18 (35)	14 (33)	49	2002	31 (51)	21 (42)	79
1984	24 (40)	22 (54)	48	2003	33 (49)	23 (41)	82
1985	21 (31)	24 (41)	43	2004	36 (52)	26 (50)	80
1986	25 (36)	31 (45)	41	2005	33 (51)	23 (43)	85
1987	22 (42)	22 (64)	49	2006	29 (46)	18 (42)	81
1988	22 (40)	21 (58)	44	2007	26 (50)	16 (40)	84
1989	18 (31)	15 (37)	40	2008	21 (48)	16 (55)	86
1990	18 (26)	15 (24)	33	2009	20 (46)	14 (48)	79
1991	22 (33)	19 (33)	43	2010	19 (47)	14 (50)	86
1992	26 (34)	22 (32)	47	2011	22 (46)	16 (36)	87
1993	30 (43)	27 (41)	54	Average	25 (40)	19 (41)	61

Table 1: **Recommended portfolio size to achieve 90% reduction in diversifiable risk on average (and 90% of the time).** The table presents the average number of stocks and the 90th percentile of number of stocks (in parenthesis) based on SD, ES at the 1% level and TWSD as risk measures.

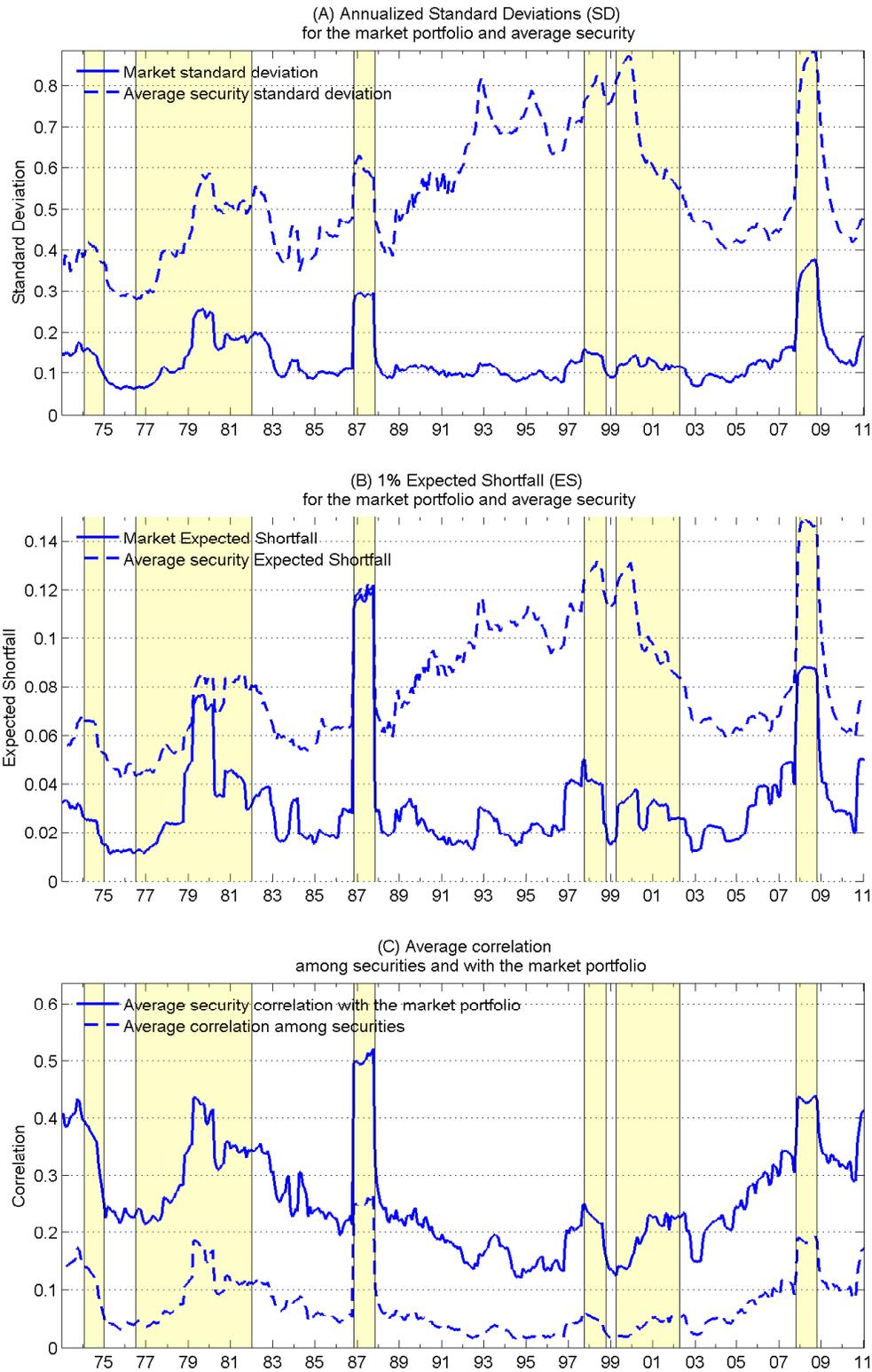


Figure 3: CANADIAN EQUITY MARKET STATISTICS 1975 TO 2011. In panel (A) the solid line shows the annualized standard deviation within each month of daily market returns based on the past 12 months' returns. The dashed line represents the average security standard deviation. Panel (B) depicts $ES_{1\%}$ of the market portfolio (solid line) and the average security $ES_{1\%}$ (dashed line). Panel (C) shows the average security correlation with the market portfolio (solid line) and the average correlation among securities (dashed line). Shaded regions in the figure represent periods of crises and correspond to events of the 1973 oil crisis (1973-1974), the 1979 oil crisis (1979-1982), Black Monday (1987), the collapse of Long Term Capital Management (LTCM) in 1998, the Dot-com bubble (2000-2002) and the Global Financial Crisis (2008).

PANEL A: NUMBER OF STOCKS REQUIRED TO ACHIEVE 90% REDUCTION IN DIVERSIFIABLE RISK IN THE PERIOD 1975-2011		
	on average	90% of the time
Based on σ	25	40
Based on $ES_{1\%}$	19	41
Based on $TWSD$	61	-

PANEL B: CANADIAN STOCK MARKET DESCRIPTIVE STATISTICS		
$\bar{\rho}_{ij}$	0.08	
$\bar{\rho}_{im}$	0.27	
$\bar{\sigma}_i$	52.3%	
σ_m	13.4%	

PANEL C: SMALL VS. LARGE PORTFOLIOS AND ASSOCIATED MARKET CHARACTERISTICS		
Recommended n based on σ	<18	>33
$\bar{\rho}_{im}$	0.36	0.18
σ_m	15.9%	11.1%

PANEL D: SMALL VS LARGE DIFFERENCE B/W AVERAGE AND 90TH PERCENTILE RECOMMENDED NUMBER OF HOLDINGS		
Δn based on σ	<11	>19
$\bar{\rho}_{im}$	0.27	0.34
σ_m	10.3%	18.7%

Table 2: **Portfolio size results.** **PANEL A** provides the average number of stocks required to diversify on average (and 90% of the time) 90% of diversifiable risk between 1975 and 2011 using the three risk measures. **PANEL B** details average correlations among individual stocks ($\bar{\rho}_{ij}$), average correlations of stocks with the market ($\bar{\rho}_{im}$), average security standard deviations ($\bar{\sigma}_i$) and market volatility (σ_m). **PANEL C** relates recommended number of stocks with market characteristics. We identify years with the largest recommended portfolio sizes (top 3rd of the sample) and estimate $\bar{\rho}_{im}$ and σ_m for these years only. We then identify years with the lowest recommended portfolio sizes (bottom 3rd of the sample) and estimate $\bar{\rho}_{im}$ and σ_m for these years only. **PANEL D** is constructed similarly to **PANEL C** but relies on the difference between the number of stocks required to assure the investor of the desired level of diversification 90% of the time and the portfolio size of the average investor (the difference between the dashed and solid lines in Figure 2).